Spherical geometry: area of triangle

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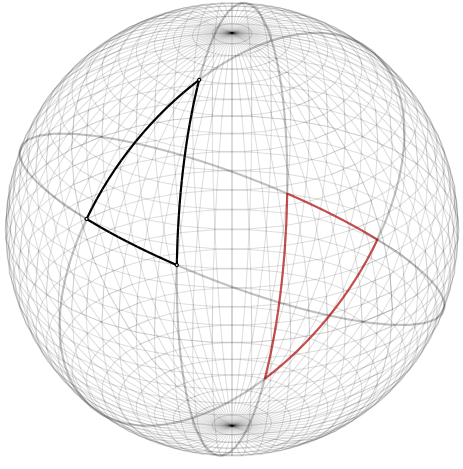
Antipodal triangle

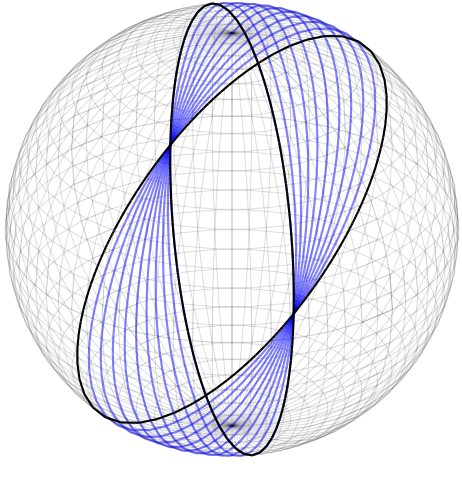
The planes (corresponding to the three great circles) that meet at three angles on the front meet in three angles at the back as well, making an equal triangle by AAA.



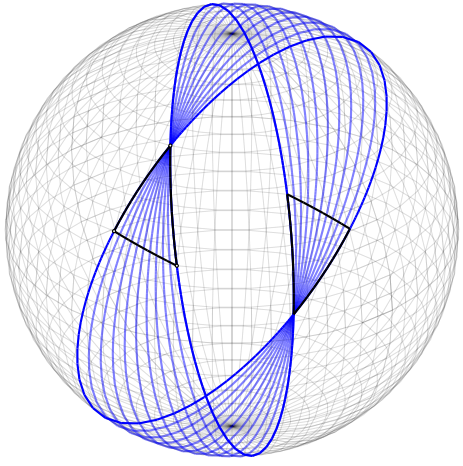
The “lunes” of a spherical angle

Area between two great circles with angle θ:

$$A(\text{lunes}(\theta)) = (2\theta/2\pi)^{\text{th}} \text{ part of the full sphere } = (2\theta/2\pi)(4\pi) = 4\theta$$



Lunes of triangle angle include triangle and antipodal triangle



Area of spherical triangle

$$\underbrace{A(\text{lunes}(a))}_{4a} + \underbrace{A(\text{lunes}(b))}_{4b} + \underbrace{A(\text{lunes}(c))}_{4c} = \underbrace{A(\text{sphere})}_{4\pi} + 4A(\text{triangle})$$

$$\implies A(\text{triangle}) = a + b + c - \pi$$