Perspective origins of projective geometry

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Parallel lines meet at infinity

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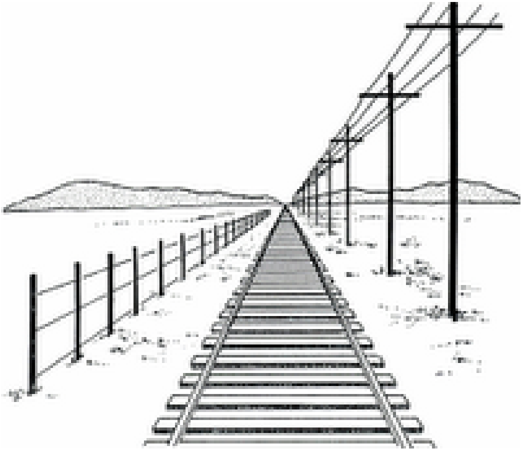
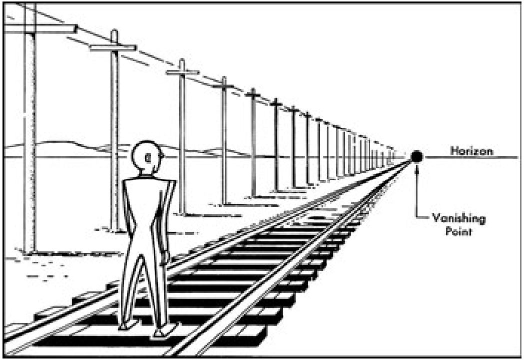
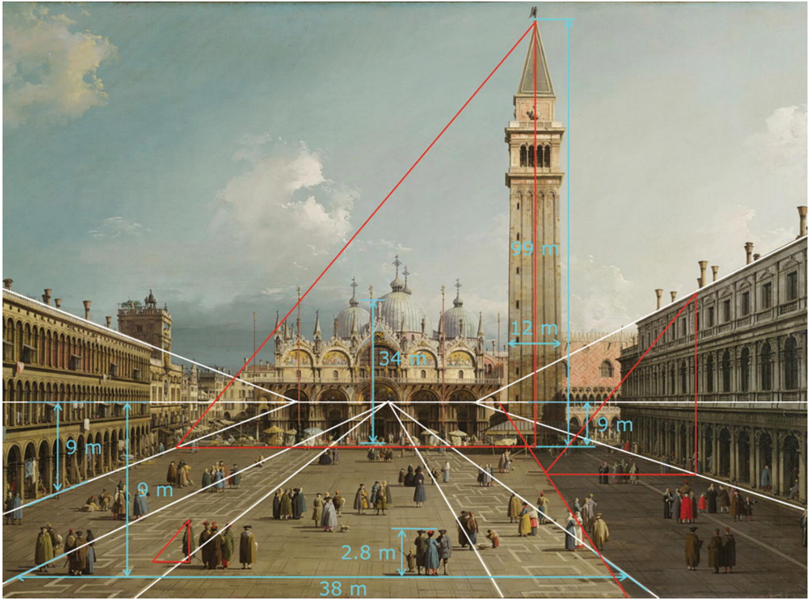


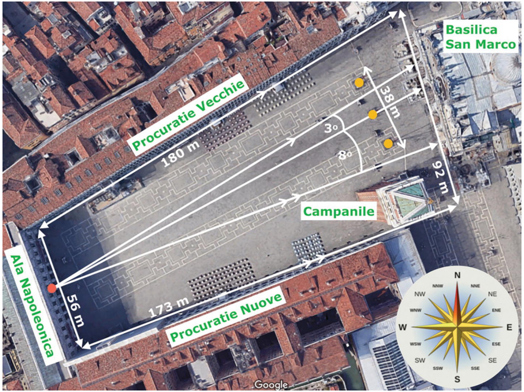


Mistake? Parallel walls have different vanishing point?



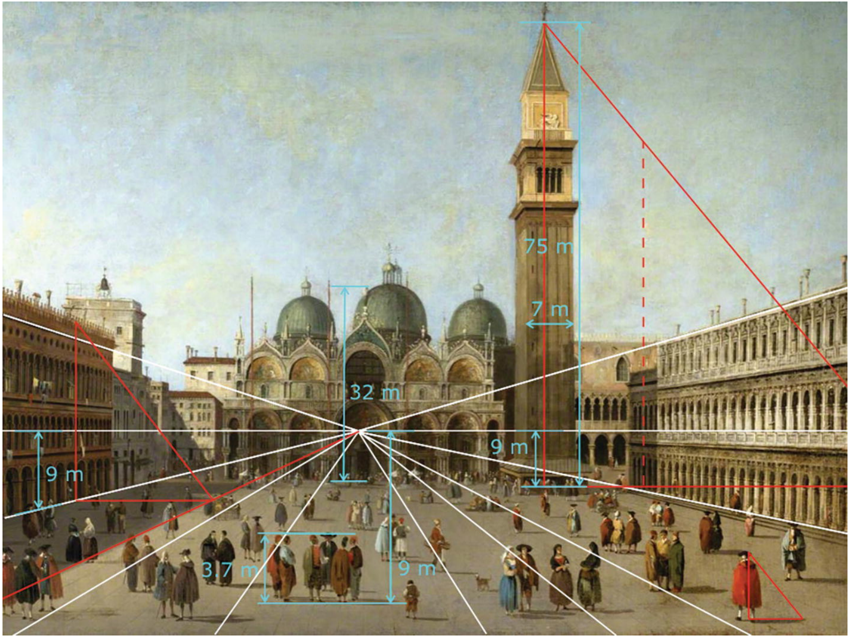
Piazza San Marco, Canaletto, 1730–1734. Fogg Museum (Harvard Art Museums), Cambridge, MA, USA.

Not a mistake! The walls are not parallel!



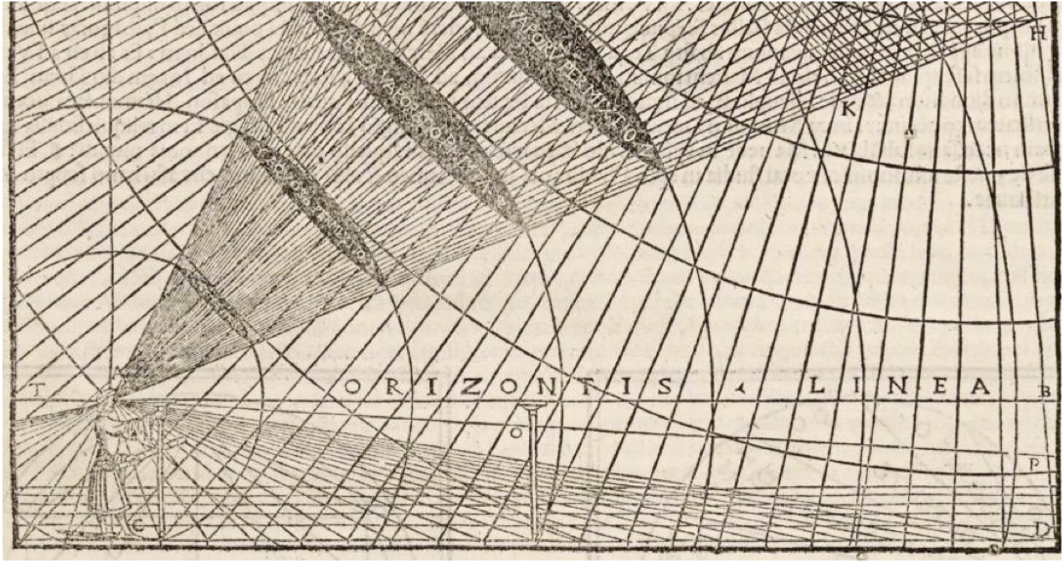
Erkelens, C. J. (2020). Perspective on Canaletto’s Paintings of Piazza San Marco in Venice, Art & Perception, 8(1), 49-67. https://doi.org/10.1163/22134913-20191131

Knockoff/forgery by artist who didn't know the geometry of the Piazza



Thirlestane Castle, Lauder, Scotland.

The horizon is perpendicularly in front of our eyes



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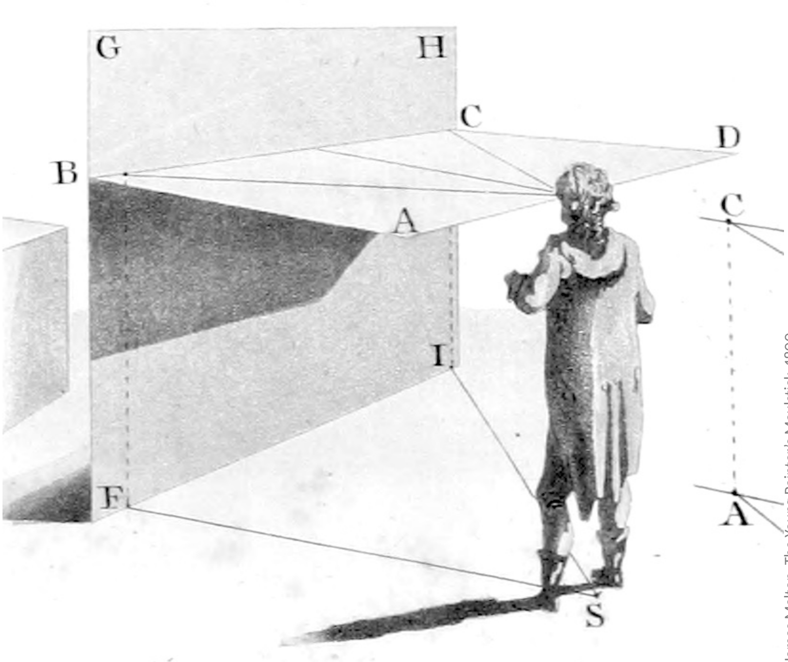


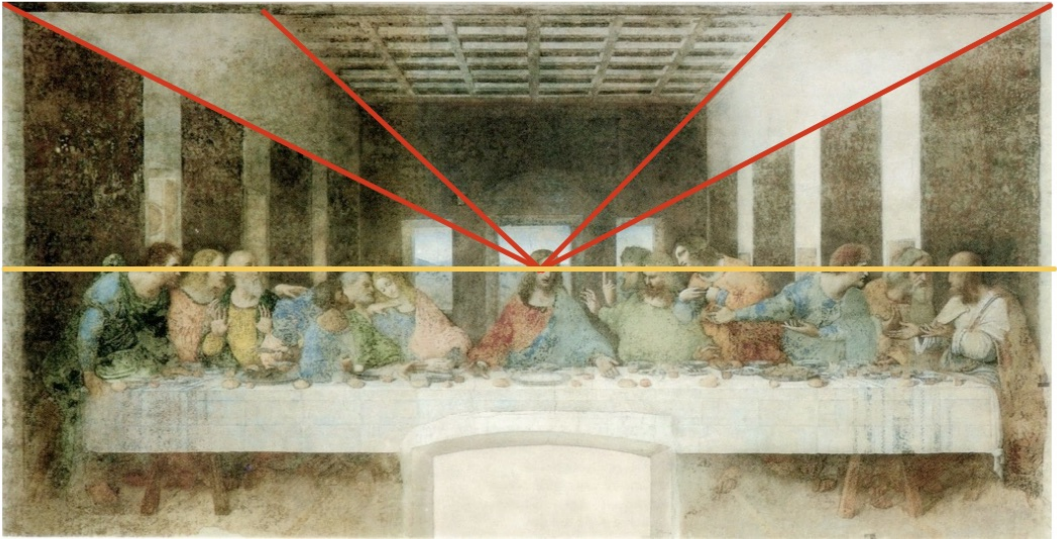
James Malton, The Young Painter's Maulstick, 1800.

❓ How tall were the photographers?

Low vanishing point used for dramatic effect

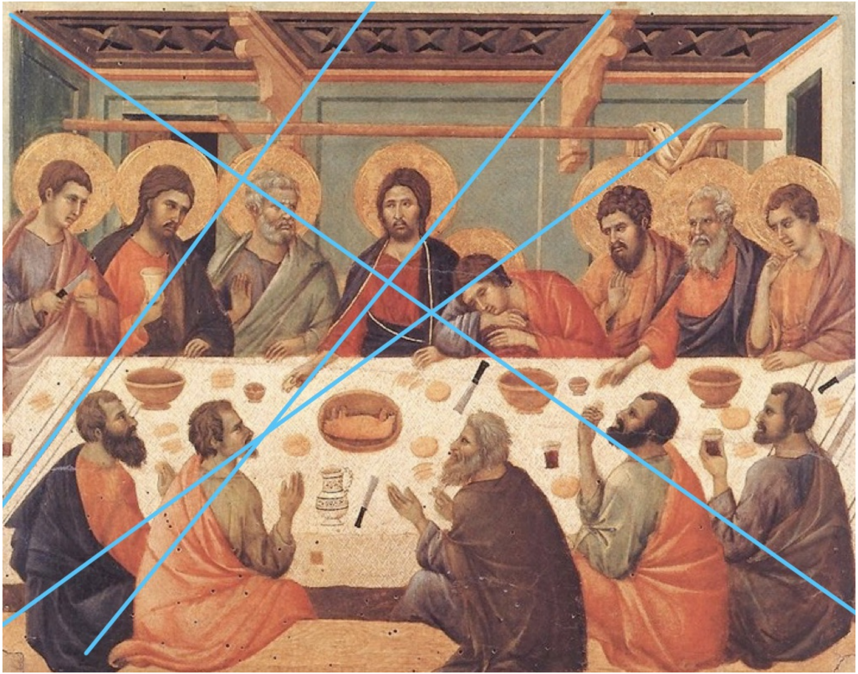
Mantegna, St. James led to Execution, 1452.

Last supper with correct perspective



Leonardo da Vinci, The Last Supper, 1495–1498.

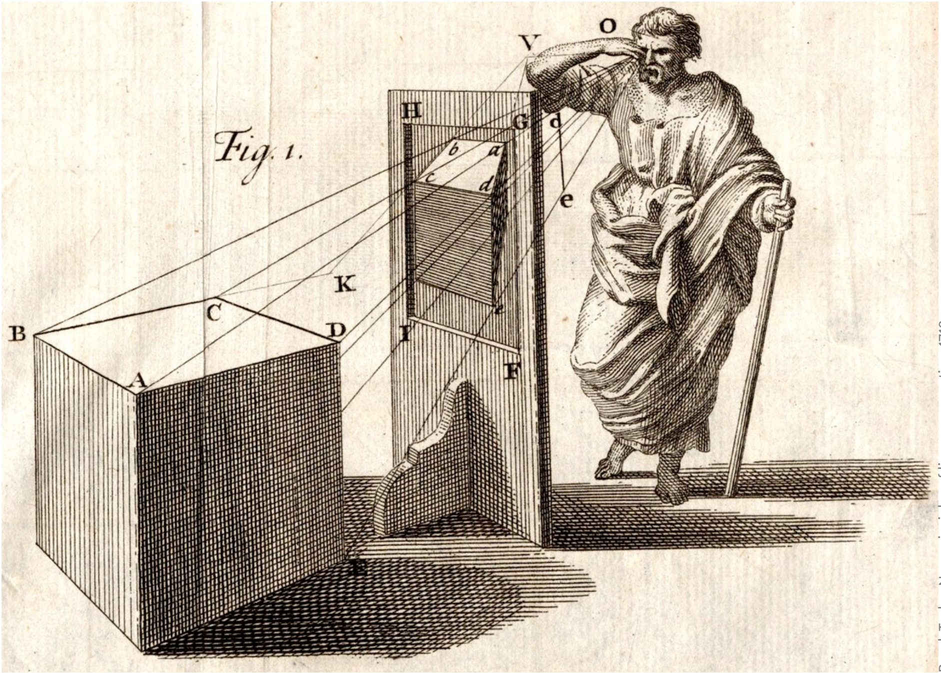
Last supper with false perspective



Duccio, The Last Supper, 1311.

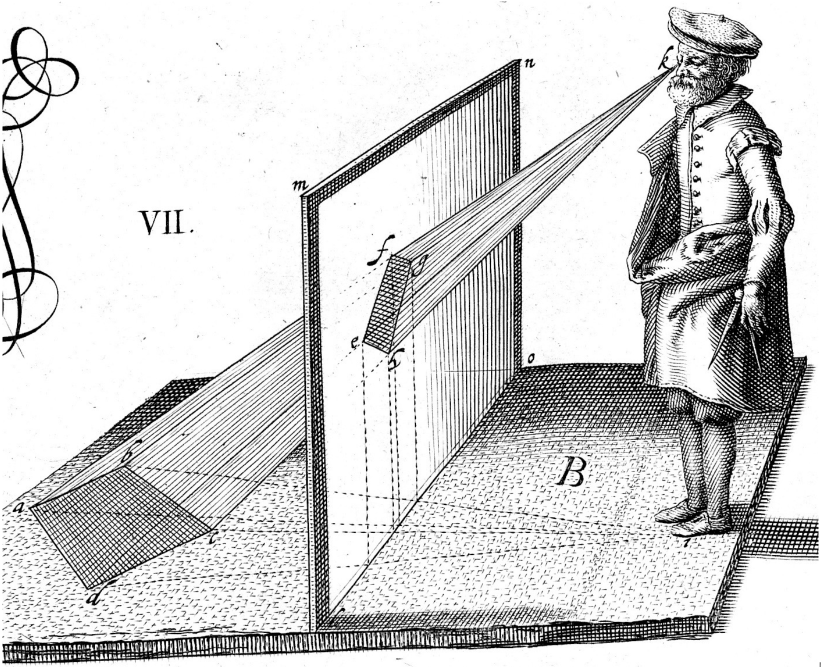
Alternative philosophy: show each item from best angle

True perspective = cross-section of “pyramid” or “cone” of rays



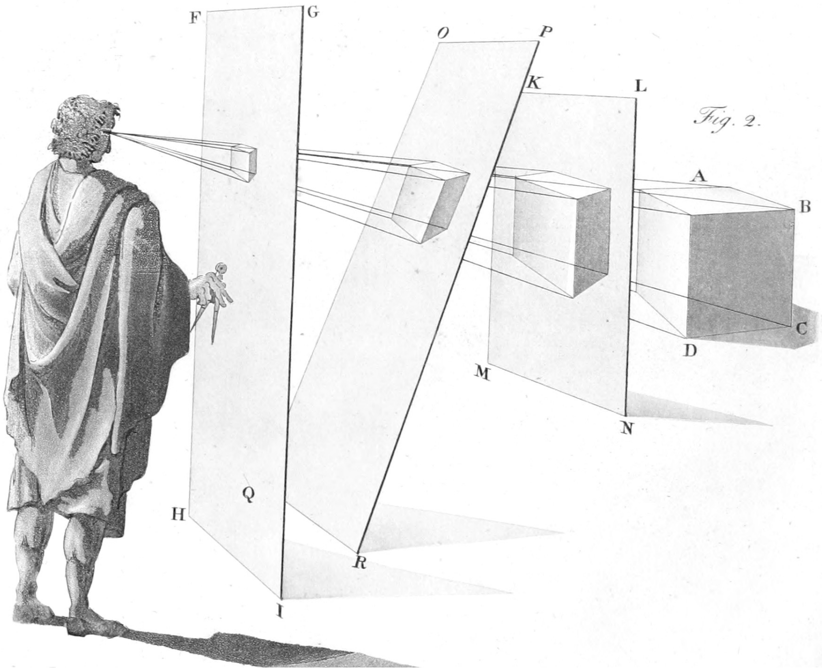
Brook Taylor, New principles of linear perspective, 1719.

True perspective = cross-section of “pyramid” or “cone” of rays



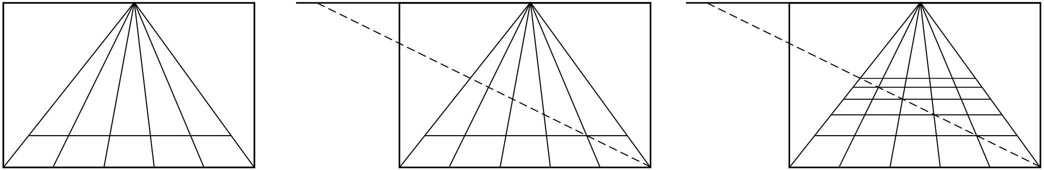
Marolois, Opera mathematica.

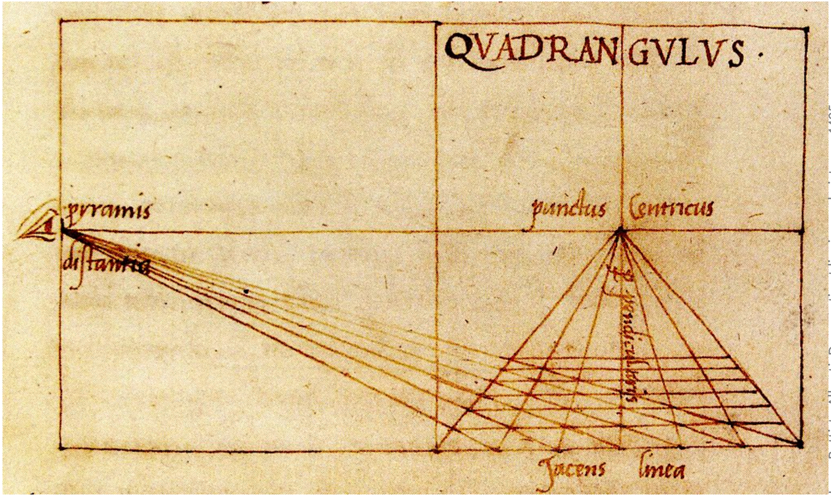
True perspective = cross-section of "pyramid" or "cone" of rays



James Malton, The Young Painter's Maulstick, 1800.

Projections line-preserving $\implies$ how to draw tiled floors



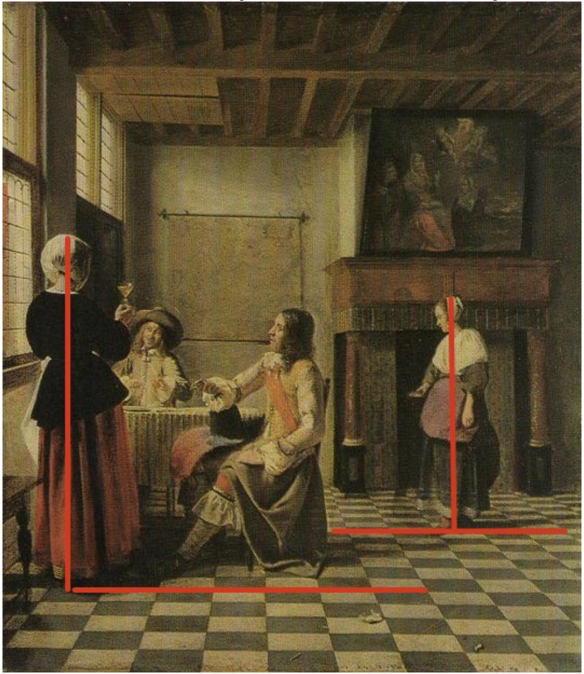


Leon Battista Alberti, De punctis et lineis apud pictores, 1435.

❓ Which of the two women is taller?

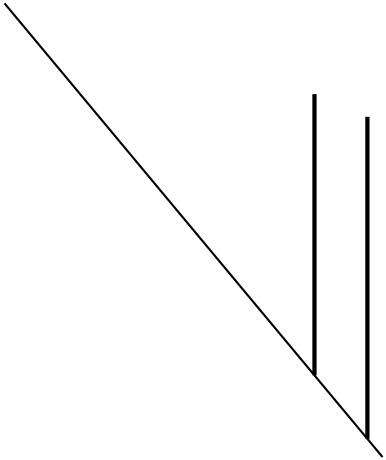
Pieter de Hooch, 1658.

Sizes at different depths can be compared via floor



Pieter de Hooch, 1658.

❓ Draw the third telephone pole

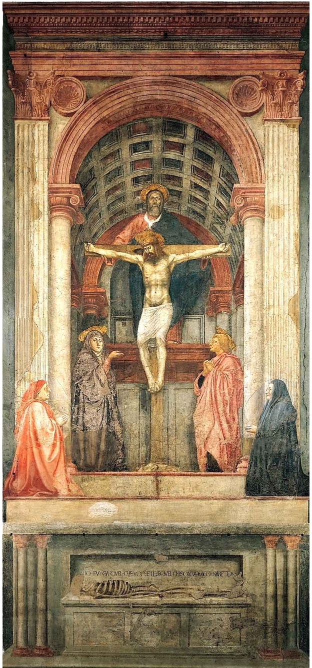


Two telephone poles standing on the side of a straight road. Explain how to draw the correct perspective view of the third telephone pole. Justify your steps.

(In reality, the telephone poles are all of the same size and equally spaced.)

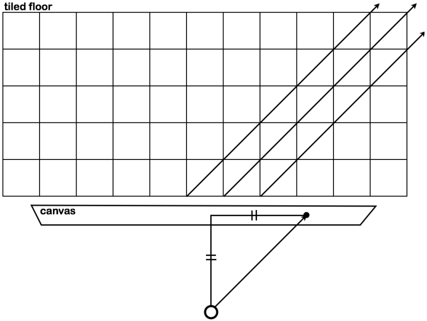
Hint: Projective questions can be answered by projective methods. That is to say: draw using only that two points determine a line, not using lengths, angles, or parallelism.

Viewing distance can be inferred from vanishing point of diagonals

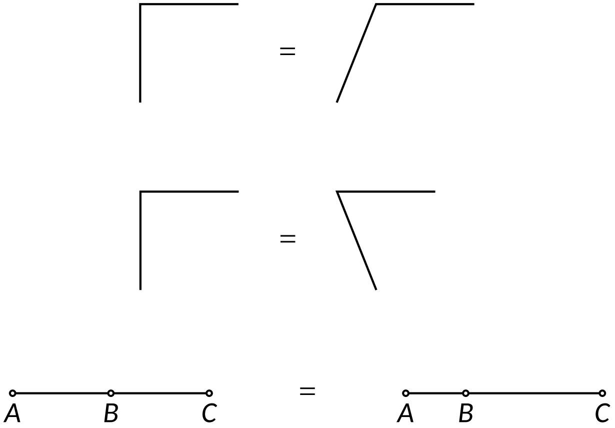


Masaccio, Holy Trinity, Florence, 1425–1428.

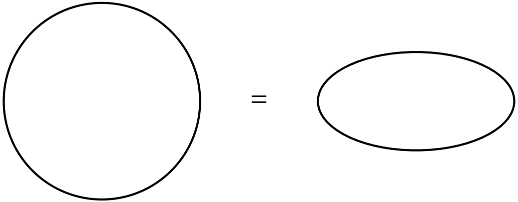
❓ Where should you stand to experience the perfect perspective illusion?

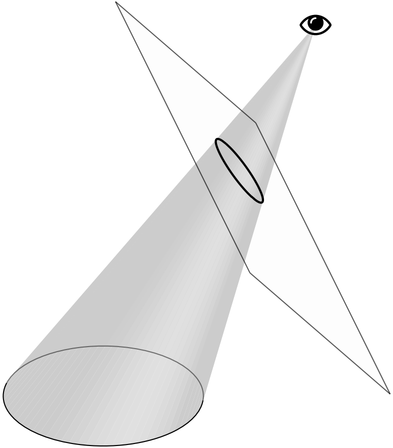


Projectively equivalent

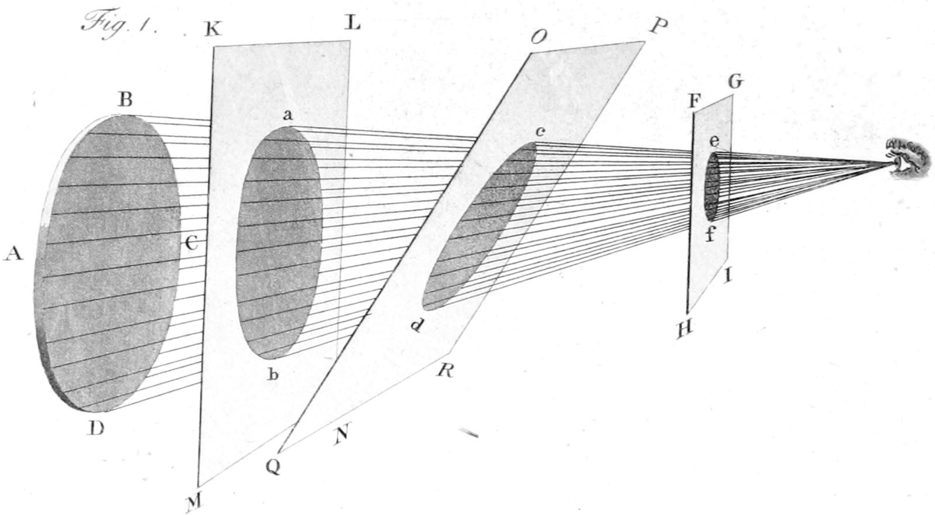


Projectively equivalent



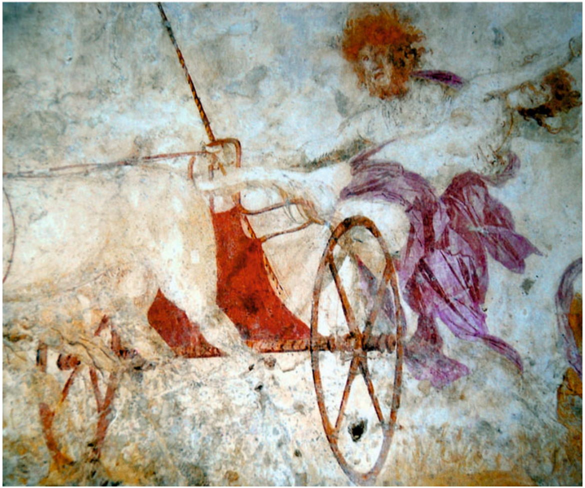


Perspective view of circle

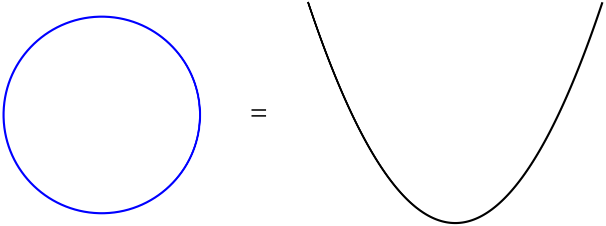


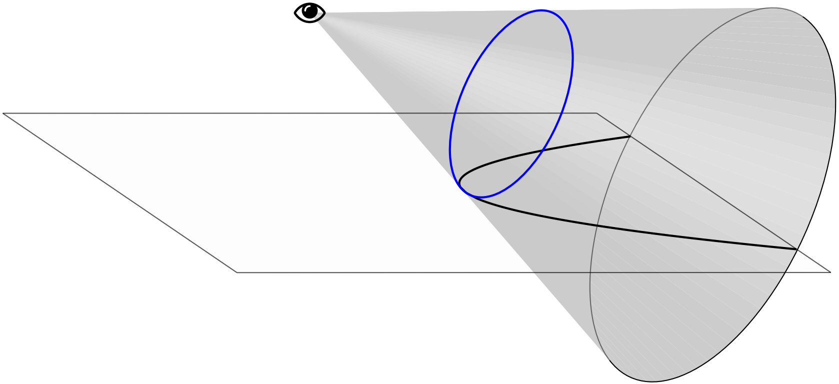
James Malton, The Young Painter's Maulstick, 1800.

Mural in Macedonian tomb (−4th century) with ellipse wheels

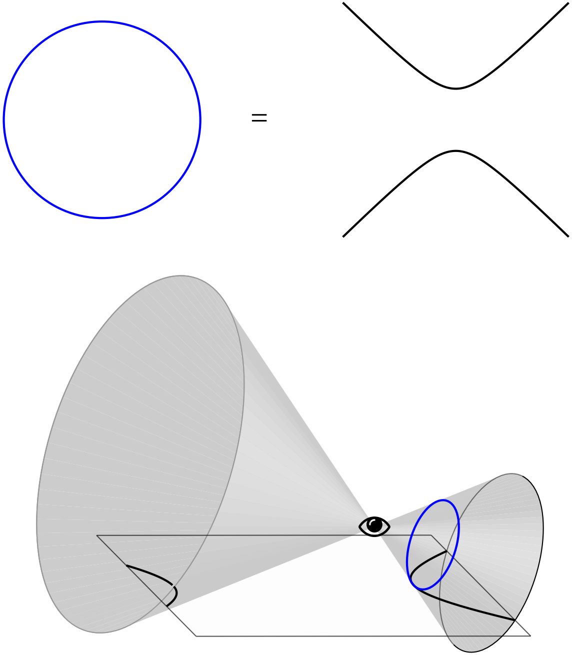


Projectively equivalent (visualisation: https://twitter.com/matthen2/status/1438171989160644614)

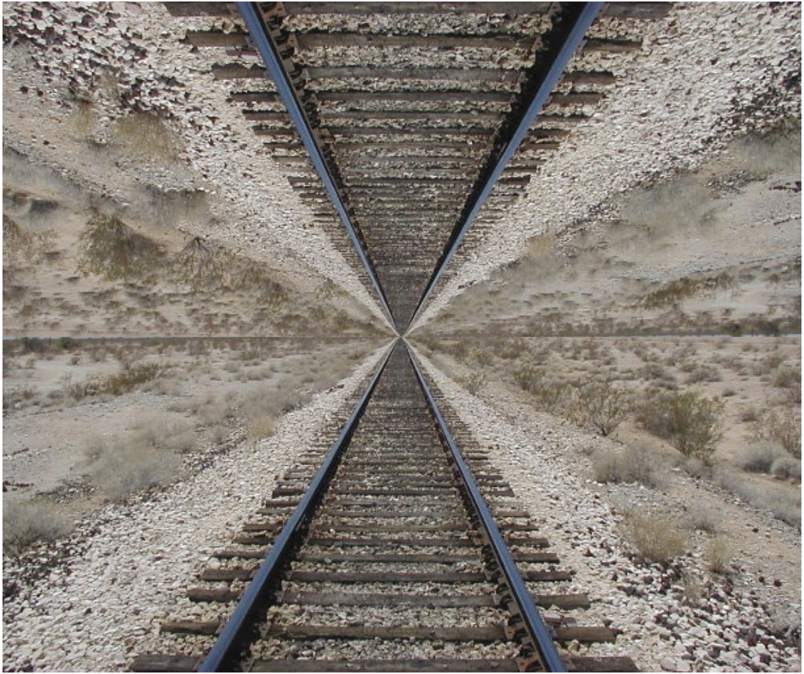




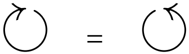
Projectively equivalent



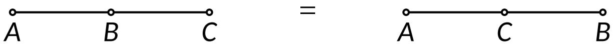
In projective geometry, we “see through the back of our heads”



❓ Projectively equivalent?



❓ Projectively equivalent?



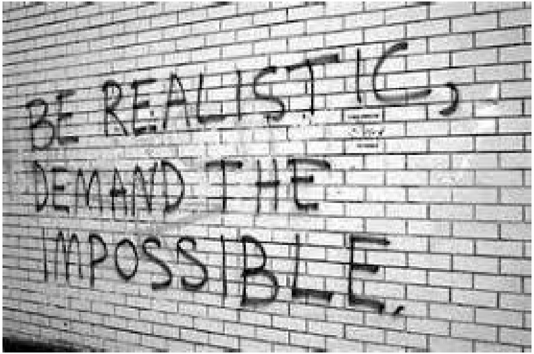
“Useful fictions”

- $\sqrt{-1}$
- dx (which is both $= 0$ and $\neq 0$, so to speak)
- (parallel line)$\cap$(parallel line)

$\mathbb{R}$ is enlarged to $\mathbb{C}$ by insisting that all algebraic equations must have roots (even if we have to make up “imaginary” ones).

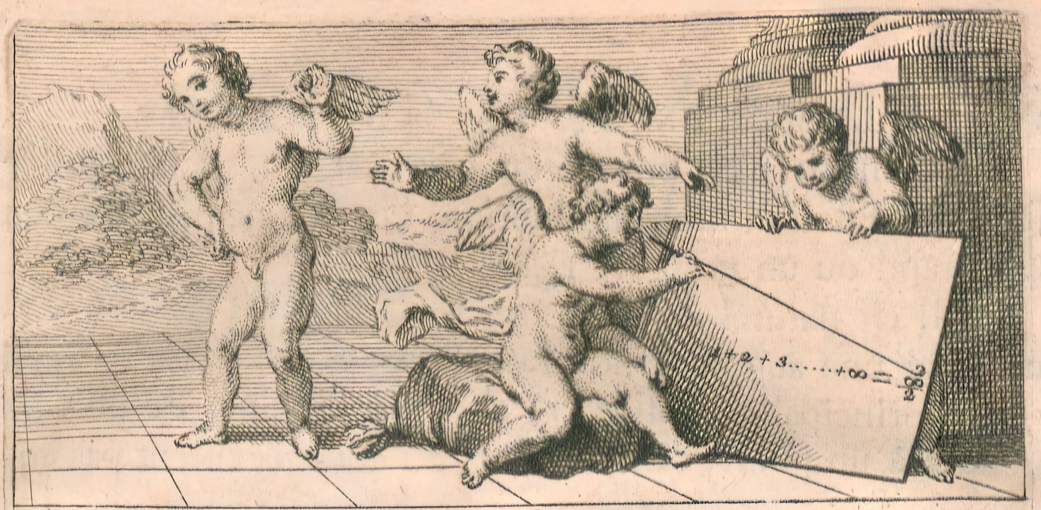
$\mathbb{E}^2$ is enlarged to $\mathbb{P}^2$ by insisting that all lines must have intersections (even if we have to make up “imaginary” ones).

Demand the impossible



"At any given moment there is only a fine layer between the trivial and the impossible. Mathematical discoveries are made in this layer." — A. N. Kolmogorov, 1943

"What is forbidden to count, he counts."



Quod numerare nefas, numerat