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by H.J.M. Bos

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1.

"I don't know", wrote Oldenburg to Huygens on the 18th of november 1676, "if you know a certain doctor Leibnitzius of Mainz", who is interested in the philosophy of nature and the properties of motion. "He pretends to have found the very principles of the rules of motion which the others, he says, have simply just given, without *a priori* demonstrations".[1] Oldenburg included a copy of the letter[2] in which this doctor Leibnitzius set out his claims: the rules of collision as presented by Wallis, Huygens and Wren, and published in the *Philosophical Transactions* of 1669, though true descriptions of the observable phenomena of collision, are much different from the abstract first principles applying to motion in vacuo or in a medium at rest; cohesion and other properties of bodies can be explained by an easy and simple argument concerning contigua and continua; the relation of these two has to be understood by very subtle arguments on the nature of the point and of indivisibles; the existence of the vacuum can be proved, and also the existence of time without motion and of incorporeal beings; this is proved with no less than Euclidean certainty, and the proof gave its author more joy than if he would have found the quadrature of the circle or the perpetuum mobile;...

This was, as far as we know, Huygens' first confrontation with Leibniz' ideas. It was a typically Leibnizian document, as Huygens was to see many more: it showed an unusual mind, it suggested deep insights, but it claimed much more than it told, promising solutions to the most fundamental questions, but deferring the results till there would be more time and space to work them out. A reaction of Huygens is not recorded. But on later occasions - as for instance in his comments on Leibniz' algebraic studies in 1675[3], in his reaction on Leibniz' draft of the Analysis situs[4] in 1679, in his reaction on Leibniz' first rather vague indications of his new infinitesimal methods[5], and in his private comments[6] in the margins of Leibniz' Acta Eruditorum articles - Huygens often showed himself annoyed by that mixture of secretiveness and promise which characterises so many of Leibniz' communications.

These reactions show a tension in the relation between Leibniz and Huygens, and it is this tension which remains most in the memory after reading the documents on their communications. The tension lies deeper than the differences in style and fields of scientific interests which I shall discuss below. It is a tension between different characters.

Huygens is a cautious man, setting high and strict standards of clarity and relevance for his scientific work, wary of grand schemes and promises without substance. Leibniz sees the value of his work as much in its promise for further understanding as in its actual content, a strategist of knowledge rather than a gatherer of facts and theories.

Despite this tension and the differences between Huygens and Leibniz, the "savant" and the "philosophe", as Gueroult characterises them in their relation,[7] their contact was a fruitful one.

2.

Two years after that first confrontation with Leibnizian writing Huygens had met the young philosopher in person.[8] Their first meeting, in Paris, must have taken place in the autumn of 1672, and we are fairly well informed on their contacts during Leibniz' Paris period.
We know of discussions on summation of series in 1672.[9] Leibniz had remarked earlier that a series whose terms can be recognized as differences of successive terms in another series, is easily summed. He told Huygens of his interest in series and Huygens suggested trying out his ideas on the sum of the reciprocal triangular numbers $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10}$ + etc. Leibniz succeeded in summing the series[10] and gained the admiration of Huygens, who told him his own method of summation for this case. Leibniz generalised his results to the summation

of the reciprocals of other combinatorial numbers (triangular, pyramidal, etc.), arranging these series in a display of rows and columns and stressing the analogy with Pascal's arithmetic triangle.
Before the end of the year he prepared a publication of these results, taking the occasion to comment on the infinites occurring in the summations of the series, but due to the break in appearance of the Journal des Savans, it was not published. Before the end of 1672 Leibniz must also have shown the plans of the arithmetical machine to Huygens, who thought the invention very ingenious.[11]
In this period Huygens was seeing the Horologium Oscillatorium through the press (the printing had started in september 1672 and the first copies were ready in april 1673)[12] and it is most likely that he talked with Leibniz on the contents of that great book.

In early 1673 Leibniz travelled to England where he was rather harshly confronted with his ignorance of current mathematical literature. His results on series appeared to be known already. Leibniz returned, determined to remedy his lack of mathematical reading. Here Huygens acted as a sympathetic guide. He presented Leibniz with a copy of the newly published Horologium Oscillatorium and naturally the conversation turned to mechanical concepts. Leibniz blundered over the concept of the centre of gravity, which earned him a reference to the works of Pascal.[13]

From Pascal's work Leibniz gained the insight in the importance of what he was soon to call the characteristic triangle, an insight which enabled him to generalise to all curves a result which Pascal asserted in the special case of the circle.(14) The result is a relation between the moment of a curve with respect to its axis and the quadrature of a second curve whose ordinates are equal to the normals of the original curve.(15) He informed Huygens, who again praised the result, told that he knew it already and gave advice on further literature, especially concerning the determination of the normal, that is, to tangent methods in general.

Leibniz now embarked on a full program of reading: Pascal, Gregory of St Vincent, Descartes, Sluse, Cavalieri, Guldin, Torricelli, Wallis, Huygens and others.

The insight in the characteristic triangle and an idea to consider an area under a curve not as divided by parallel ordinates into thin strips, but by radii from the origin into thin triangles, gained Leibniz his first great result: the transmutation rule,(16) which relates the area under a curve to the area under a second curve derivable from the first through constructing the tangent at each point. He soon convinced himself of the wide applicability of this transformation; it yielded a general proof for the quadratures of all higher parabolas and hyperbolas,(17) applied to the cycloid it gave a beautiful, be it special, result, and above all, applied to the circle, and in combination with series expansion through long division as Leibniz had learned from Mercator's Logarithmotechnia, it yielded a most impressive result: Leibniz' series for π , $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \text{etc.}$

These results are from 1673, but it seems that, unlike the case of summation of series and the characteristic triangle, Leibniz did not rush immediately to Huygens to tell all the details. In fact it lasted till october 1674 before Huygens saw the proofs of the results[18] (in summer 1674 he had received a summary of Leibniz' results in geometry) on which he commented very positively in the first letter we have from him to Leibniz (7-11-1674)[19].

Still, Huygens and Leibniz did discuss the quadrature of the circle in 1673; on the thirtiest of december of that year Huygens lent to Leibniz James Gregory's Vera circuli et hyperbolae quadratura and his own De circuli magnitudine inventa, with the request to go over Gregory's claim to have proved the impossibility of the quadrature - a claim which Huygens did not accept but could not refute. The subject kept Leibniz' interest throughout 1674[20] and led him to study the works of Gregory more closely and to acquire, through these, a further thorough training in infinitesimal mathematics, finding that most of the methods he had worked out for himself were already known by others.

After 1674 the connection between Leibniz and Huygens becomes less intensive. Partly this may be because Leibniz turned his interest to algebra which did not fascinate Huygens very much. From december 1674 dates Leibniz design for the circinus aequationum[21], an instrument to solve algebraic equations.

Leibniz pursued algebraic studies in the first half of 1675 and sent summaries to Huygens who went through them. A similar summary was sent in the summer, but now Huygens did not react and Leibniz had to press with a further request and a new summary. Finally Huygens answered in september with a polite but not very enthusiastic letter:[21a] he misses proofs of Leibniz' two central assertions, he finds the results partial, Leibniz'findings on the use of imaginaries in algebra are surprising but there seems to be something below the surface which still needs explanation, and as to the algebraic machine - it is not difficult to read between Huygens polite lines that he doubts its effectiveness.
Meanwhile Tschirnhaus had arrived in Paris and Leibniz found that even with his (Tschirnhaus') greater proficiency in algebraic manipulations not much more seems to be achievable in the direction of general methods for the solution of equations.[22] So we find him shifting his interests back to questions in the geometry of curves.

In the mean time Leibniz had been a close witness to Huygens' presentation of his invention of the spiral balance for clocks; on 22 january 1675 Huygens had informed him about the discovery, on which patents were granted within some monthes and concerning which a controversy arose with Hooke. The discovery induced Leibniz to publish a related invention of his own concerning clock regulation, in which Huygens showed interest.[23]

Even if Leibniz had intended to be more open to Huygens with the researches he started in autumn 1675 and which were to contain the invention of the calculus,[24] circumstances would have prevented that; Huygens fell ill towards the end of 1675 and could not receive visits. The only direct contact between the two in the last year of Leibniz' Paris period seems to have been an exchange of short letters concerning Leibniz' wish to secure himself a position in the Académie; just before his departure to Holland for recovery in june 1676, Huygens advocated Leibniz' wishes with Galois and wrote to Leibniz about this.[25]

In 1679 Leibniz wrote again to Huygens.[26] Some letters were exchanged; Leibniz sent a resume of his ideas on Analysis situs[27] and an example of a tangent problem solved by his new method for tangents,[28] but Huygens reacted rather coolly and the correspondence dropped. It was taken up again in 1688 on occasion of Huygens' solution of a problem publicly proposed by Leibniz.[29] The correspondence soon became intensive and remained so till Huygens' death. Leibniz gave indications of his new results and methods, asked Huygens' comments, let him present questions to test his own new method for inverse tangent problems, discussed questions in mechanics and many other topics.
Clearly the relation has now changed, Leibniz has matured, he is still interested in Huygens' judgement and critique but does not seem to learn from him; rather he is bent to convince Huygens of the value of his inventions.

If we look for Huygens' influence on Leibniz we must not look to the later correspondence but to the contacts during Leibniz' formative years: the Paris period.

3.

What, then, was Huygens' influence on the formation of Leibniz' ideas?

Although we are much better informed on the contacts between Huygens and Leibniz on mathematical matters, it is not there, but rather in Mechanics, that Huygens' influence is most tangible.(30) If Leibniz learned from Galileo the importance of the concepts of continuity and infinitesimals in mechanics, and if he took from Descartes the idea of a conservation law, it was from Huygens that he took over the whole mathematical and kinematical structure on which he was to work out his own dynamical ideas: the three conservation laws (of respective velocity, of the vectorial sum of motions and of the sum of the products mv^2), the principles concerning the centre of gravity and the impossibility of perpetual motion, the three paradigm processes collision, fall, free or along inclined planes, and horizontal uniform motion, and the relations between these processes.

In offering this structure of kinematics and its relation to experimental situations, Huygens' influence induced Leibniz to take the <u>a posteriori</u> side of mechanics seriously, which saves his later work from the over-abstraction of his previous <u>a priori</u> studies.

Thus Huygens supplied the weapons with which later Leibniz was to attack Cartesian dynamics, of whose insufficiency Huygens had already convinced himself.

However, Leibniz uses all this material in his own way; he is more attentive to inner mathematical coherence of the principles, which leads him to incorporate a fourth conservation law, that of <u>actio</u>. He applies his infinitesimal concepts and his calculus to clarify the dynamical concepts and to make them efficient.
But there is a deeper difference between Huygens' and Leibniz' approaches to mechanics. Whereas Huygens builds his system as exclusively kinematical as possible, trying to reduce the role of dynamical concepts and of metaphysics to a minimum, it is precisely in the direction of dynamics and metaphysics that Leibniz' interests lie. The conservation laws, which Huygens considers primarily as mathematical results and which he hesitates to take as basic axioms for his system, these laws indicate for Leibniz that there exist dynamic entities (as live force) and that these are the proper object of study in mechanics. Here he draws conclusions from the Huygensian material in which Huygens does not want to follow him. Both accept that direct contact is the only possibility for the exchange of motion, but Leibniz' further conclusions on continuity, on the elasticity pervading all matter, on the infinite divisibility and on the non existence of a vacuum are contrary to Huygens' mechanistic view of reality. In the

discussion on these matters in the letters of the 1690's Huygens does not offer much in the way of argument, but seemingly trusts his feeling more: if Leibnizian logic would induce him to conclusions difficult to visualize and without fruitful use, Huygens rather adheres to a simpler world view of particles of matter, completely hard and with a wide variety of sizes, moving in a vacuum. This world view does not explain all phenomena, but it has proved itself useful (as for instance in the theory of refraction and double refraction).

So, in the case of mechanics, Huygens' influence on the formation of Leibniz' ideas is that of a supplier of material and structure, of a firm basis on which Leibniz erects a whole further building of dynamical and metaphysical concepts and arguments.

4.

The case of mathematics is less straightforward. The reason, of course, is that the mechanics to which Leibniz was to contribute was a science pursued by few only, of which Huygens was by then the greatest, whereas mathematics was pursued by many.

Huygens introduced Leibniz to mathematical literature, stimulated his mathematical interest in personal discussions, suggested problems, praised Leibniz' earliest independently found results and set the example of thorough mathematical work. But it is not possible to pin down important mathematical results or methods which Leibniz learned directly from Huygens.

In particular, the key ideas which he combined in his discovery of the calculus in 1675, were Leibniz' own (be it that others had hit on them before him)[31]. These ideas were: first the insight that summing of sequences and taking their differences are inverse operations and that similarly determining quadratures and tangents of curves are inverse operations; secondly the recognition of the crucial role of the characteristic triangle in finding transformations of quadratures; and thirdly Leibniz' interest in symbolism and notation, in connection with his idea of a *characteristica generalis*, a general symbolic language in which an art of invention would be codified so as to yield almost automatic processes of invention. It is well documented that the first two of these ideas were discussed between Huygens and Leibniz, but in both cases Leibniz brought them in the discussion and Huygens praised them as important. Also the third idea will have been brought up by Leibniz in discussions with Huygens. But Huygens was averse to general statements which promised more than they gave, and he did not see the elaboration of new notations and new rules of calculation as likely ways to gain new power over problems of quadratures, tangents, inverse tangents etc.

So we see Huygens' influence in the case of mathematics restricted by differences in interest and style. Huygens was not interested in number theory and not very much in algebra, both of which arrested Leibniz' universal interest in the years 1672-1676. Also Huygens' style was not the search for grand schemes and general solutions, not the elaboration of analytical kinds of calculus or automatised arts of invention. His mathematics is organised by problems not by methods.

Something more has to be said about the difference in mathematical style between Huygens and Leibniz, and in particular on Huygens' preference for rigourous Archimedean methods. The Archimedean rigour in Huygens' mathematics should not be exagerated.(12) It is true that he presented his great results, as for instance the tautochronism of the cycloid, with a scrupulously classical proof. But this does not characterise his mathematical activities as a whole. He accepted Fermatian procedures for the determination of tangents, dividing freely through small quantities which at the end are taken to equal zero. In studying quadrature problems he worked with infinitely small strips. In all his extensive studies on the catenary, for instance, or on motion in resisting media, there is no question of rigourous Archimedean methods of proof - as indeed there could not be because Huygens' intention there was to find the solution of problems, whereas the Archimedean method only serves to prove that the found solutions are correct.
So it was not so much in rigour of proof that Huygens and Leibniz differed - both would agree that the Archimedean way

was the final answer as to reliability - but in the style of the methods of invention: for Huygens these were strongly geometrical, the algebraic notation only serving to describe what was going on in the figure, and more craftmanlike, in the sense that he was not over much interested in abstracting general methods from the solutions of the problems at hand. For Leibniz, this was the essence of the program, he was interested in the problems only as far as they illustrated the use of methods. He saw in Descartes' analysis the possibility to raise above the restrictions of the geometrical figure, and he endeavoured to extend the Cartesian analytical tools to those realms (of transcendental curves) which Descartes had excluded from geometry because his analysis appeared not to cover them.

These differences in interest and style must have come out in the contacts between Leibniz and Huygens in Paris, and it is a tribute to Huygens' qualities as a teacher and judge of intellect that, despite the differences, he saw the qualities of the young mathematician and gave him the introduction to higher mathematics for which Leibniz has always remained grateful.

5.

Huygens' influence on the formation of Leibniz' ideas was conditioned by the circumstances of their contact and the differences in style of intellectual activity between the two.

The influence was considerable, but - not surprisingly in the case of two so original minds - it is difficult to capture it. There was the influence of the supplier of facts, methods and ideas, as evident particularly in the formation of Leibniz' ideas in mechanics. There was the influence of the teacher, the great man impressed by the power of his young disciple and lending his personal experience and advice to Leibniz' "growth to mathematical maturity". There was the influence of prestige and example, shown in Leibniz' evident esteem for Huygens' critique , the value he attached to the contact and his hope for Huygens' good judgement, despite their differences in approach and style. And finally there was the influence of intense scientific discussion in general, an influence which remained also after the relation had ceased to be that of teacher to pupil, and had occasionally even been reversed, as when Huygens yielded somewhat to Leibniz' ideas in dynamics, or took pains to work himself into Leibniz' new calculus.

The latter point occasioned Leibniz to express (in a letter which was probably not sent, as Leibniz learned of Huygens' death shortly after writing it) his esteem for Huygens greatness, his respect for the different approach and his thankfulness for the influence on the formation of his ideas: "...vous, Monsieur, qui aviés toutes les raisons de monde de vous tenir entierement à vos propres methodes qui vous avoient servi à tant d'importantes decouvertes avant que j'avois commencé d'y avoir quelque entrée; et qui n'avés pas laissé de vous abaisser tout grand Maistre de l'art que vous estes, à employer encor une nouvelle Methode d'un de vos disciples, car vous ne devés pas ignorer que je pretends à l'honneur de l'estre, et que j'en ay fait profession publique plus d'une fois."[33]

Notes

I use the following abbreviations:

<u>Hofmann</u> HOFMANN, J.E. <u>Leibniz in Paris 1672-1676. His growth to mathematical maturity</u>, Cambridge, 1974; this is a translation, revised by the author, of <u>Die Entwicklungsgeschichte der Leibnizschen Mathematik während des Aufenthaltes in Paris (1672-1676)</u>, München, 1949, references are to the english edition.

HO HUYGENS, Christiaan, <u>Oeuvres complètes</u> (22 vols), the Hague, 1888-1950.

LMG LEIBNIZ, G.W. <u>Mathematische Schriften</u> (7 vols., ed. C.I. GERHARDT), Berlin and Halle, 1849-1863; reprint Hildesheim, 1961-1962.

1. Oldenburg to Huygens 18-11-1670, HO <u>7</u> 46-47.
2. Leibniz to Oldenburg nov. 1670, HO <u>7</u> 48-50.
3. See below, note 21a.
4. Leibniz explained his idea of an <u>analysis situs</u> in an appendix to his letter to Huygens of 8-9-1679, HO <u>8</u> 214-219 (the appendix 219-224). The <u>analysis situs</u> is a calculus based on the relation of congruence, and using congruence equations to characterise spheres, planes, circles, straight lines and points in space. (It has nothing to do with topology.) See FREUDENTHAL, H. "Leibniz und die Analysis Situs" <u>Studia Leibnitiana</u> <u>4</u> (1972) 61-69. Huygens answers (Huygens to Leibniz 22-9-1679, HO <u>8</u> 243-245): "mais pour vous l'avouer franchement je ne concois pas, pas ce que vous m'en estalez, que vous y puissiez fonder de si grandes espérances."
5. See for example the letters Leibniz to Huygens 25-7-1690, HO <u>9</u> 448-452 and Huygens to Leibniz 24-8-1690, HO <u>9</u> 470-472. In fact, Huygens eventually learned more about the differential calculus from his correspondence with l'Hôpital than from that with Leibniz, which shows that often the disciples are better teachers than the master.
6. See HO <u>22</u> - . After Huygens' death Johann Bernoulli saw his <u>Acta Eruditorum</u> copies, transcribed the marginalia and sent them to Leibniz. Some characteristic examples of these marginalia are: "Quam misere obscura haec omnia" (p. 796, comment on Leibniz' "De linea isochrona..." <u>Acta Erud.</u> april 1689, LMG <u>5</u> 234-237), "Speciosus titulus in re nihili", "Haec omnia intellectu difficillima et cum intellexeris nequaquam effici possunt" (p. 809, both comments on "Supplementum Geometriae Dimensoriae.." <u>Acta Erud.</u> april 1693, LMG <u>5</u> 294-301).

7. GUEROULT, M. Dynamique et Metaphysique Leibniziennes suivi d'une note sur le principe de la moindre action chez Maupertuis, Paris, 1934, p. 89.

8. The main sources concerning the relations between Huygens and Leibniz in Paris are: the various references in HO (esp. vol 7), to be found via the index s.v. Leibniz, three Leibnizian documents on the contacts, namely, a remark in "De solutionibus problematis catenariae..." Acta Erud. sept. 1691 (LMG 5 255-258), a draft of a postscript to a letter to Jakob Bernoulli april 1703 (LMG 3 71, the postscript was not sent off), and Leibniz' 1714 account of his discovery of the calculus "Historia et Origo calculi differentialis" LMG 5 392-410; relevant quotations from these three sources can be found in HO 7 245-247, note. In Hofmann these and other sources are used to give a very detailed reconstruction of the contact between Leibniz and Huygens; I have drawn extensively on this reconstruction.

9. Cf. Hofmann, ch. 2, pp 12-22.

10. Using index notation Leibniz' idea can be summarised as follows: if $a_1 = b_1 - b_2$, $a_2 = b_2 - b_3$, $a_i = b_i - b_{i+1}$, then $a_1 + a_2 + \dots + a_n = b_1 - b_{n+1}$, so that the a-series is easily summed; if b_i tends to zero we have even $a_1 + a_2 + a_3 + \text{etc.} = b_1$. For the triangular numbers 1, 3, 6, ... $\frac{i(i+1)}{2}$ Leibniz found that the reciprocals $\frac{2}{i(i+1)}$ satisfy the equality $\frac{2}{i(i+1)} = \frac{2}{i} - \frac{2}{i+1}$, so that $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \dots + \frac{2}{n(n+1)} = 2 - \frac{2}{n+1}$, and $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \text{etc.} = 2$.

11. Huygens to Oldenburg 14-1-1673, HO 7 242-244, esp. p. 244.

12. See HO 22 675. Vollgraff suggests there that this may explain an inconsistency in Leibniz' account of his contacts with Huygens. In "Historia et Origo" (see note 8) Leibniz says to have received already in 1672 a complimentary copy of the Horologium Oscillatorium. This is impossible, but Vollgraff thinks that by 1672 Huygens may have given proof sheets to Leibniz.

13. Hofmann, 47-48.

14. The result is in Pascal's Lettres de A. Dettonville..., Paris 1659, see Hofmann 48.

15. In modern symbolism (or indeed Leibnizian symbolism, but not yet conceived in 1673) the characteristic triangle dx, dy, ds, is similar to the triangle formed by ordinate y, subtangent t and normal n, hence $y\,ds = n\,dx$, so that $\int y\,ds = \int n\,dx$, which gives the relation in formula.

16. Hofmann 54-62

17. That is, curves with equations $y^q a^p = x^p b^q$, and $x^q y^p = a^{p+q}$ respectively.

18. Hofmann 63.

19. Huygens to Leibniz 7-11-1674, HO 7 393-395.

20. Hofmann ch. 6, pp. 63-78.

21. Hofmann describes Leibniz' draft of the instrument in "Über frühe mathematische Studien von G.W. Leibniz", Studia Leibnitiana 2 (1970) 81-114, esp. pp 101-104.

* 21a

22. Hofmann ch. 11, pp. 143-163.

23. Hofmann ch. 9, pp. 118-125; the article of Leibniz was published in the Journal des Savans 25-3-1675.

24. Hofmann ch. 13, pp. 187-201.

25. Leibniz to Huygens june 1676, HO 22 696, Huygens to Leibniz june 1676, HO 22 696.

26. Leibniz to Huygens 8-9-1679, HO 8 214-219.

27. See note 4.

28. Leibniz to Huygens 26-1-1680, HO 8 267-268, the example appears in an appendix, ibid. 269-271.

29. Leibniz to Huygens january 1688, HO 9 257-259. The problem was that of the linea isochrona, cf. LMG 5 234-243.

30. On Huygens' influence on Leibniz' mechanics see Gueroult op. cit. note 7, in particular pp. 82-109, and COSTABEL, P. Leibniz et la Dynamique, les textes de 1692, Paris, 1960, in particular pp. 9-14. On Huygens' mechanics in general see WESTFALL, R.S. Force in Newton"s Physics, New York, 1971, especially ch. 4, pp. 146-193, and BOS, H.J.M. "Huygens, Christiaan" Dictionary of Scientific Biography, New York, 1968- , vol. 6 597-613.

31. Cf. my account of Leibniz' discovery in Unit C3 "Newton and Leibniz" pp. 35-46, of the Open University course "History of Mathematics, Origins and Development of the Calculus" AM 289 C3, Milton Keynes, 1974 (the course consists of five units, written by M.A. Baron (units 1, 2 and 3-Newton) and H.J.M. Bos (units 3-Leibniz, 4 and 5)). In that account I have worked out the key ideas and their role in Leibniz' discovery more in detail.

32. The final summing up of Huygens' influence in Hofmann 299 seems to contain such an exaggeration. Unfortunately, the text is ununderstandable at the crucial point, where the (literally correct) english translation reads: "his bent was (...) to the pure, geometrical chain of argument which starts from a single finite entity and proceeds, by application of the indirect Archimedean method, to a result on infinitesimals". But the Archimedean method never leads to results on infinitesimals (Possibly the words finite and infinitesimal have been interchanged, but even then the statement seems not to describe the Archimedean method.)

33. Leibniz to Huygens 1-7-1695, HO 10 714-718, esp. pp. 716-717.
* 21a. Huygens to Leibniz 30-9-1675, HO 7 504-506.

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