Introduction

The bicentennial of Gauss' birthday (30.IV.1777) was commemorated at the Mathematical Institute of the Utrecht State University on April 28 and 29, 1977.
Four lectures were given, each dealing with aspects of the life and the scientific work of Gauss, and with later developments in mathematics to which his achievements gave rise.
This volume contains the texts of these lectures.

A.F. Monna.

GAUSS 1777-1855

by

H.J.M. Bos

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1. Gauss died in 1855. In that year King George V of Hannover had a medal struck in his honour on which the words appeared **"mathematicorum princeps"**, (king of mathematicians). You may know the saying about history: history is for those who would like to sit at the king's table but are not important enough to do so; history allows them to sit at the tables of kings in the past. Before I begin this sketch* of Gauss' life and work I must confess that although Gauss is one of the kings of the past, I have not been able to sit at his table. In preparing this talk I have found Gauss to be almost unapproachable, for me he always remained a remote figure. With no other figure in the history of mathematics have I felt this so strongly.

What are the reasons for this? First of all, of course, Gauss' mathematical and scientific genius. His collected works are in twelve volumes, which is impressive, but one might think that all that could at least be comprehended bit by bit (although Dirichlet once said that he found new things every time he read Gauss' **Disquisitiones Arithmeticae**). But the really astounding thing about Gauss is how much mathematical knowledge he gained and developed at a very early age and completely on his own; in how wide a field of knowledge he could see connections between apparently unconnected subjects. And, above all, it is surprising how long he kept all this knowledge to himself.

For instance, in 1811 Bessel asked Gauss how an integral should be interpreted in the case where the independent variable takes

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*) This is the text of a talk given on 28 April 1977 at the Mathematical Institute at Utrecht on the occasion of the 200th anniversairy of Gauss' birth. The data which I have used in this biographical sketch can be found in the standard biographies of Gauss: C.W. Dunnington, **Carl Friedrich Gauss, Titan of Science**, 1955, T. Hall, **Carl Friedrich Gauss**, 1970, **and** H. Wussing, **C.F. Gauss**, 1976. I am indebted to Mrs. S.M. McNab for stylistic advice.

complex values. Gauss wrote back that he had already studied this question many years before, and gave a full and clear explanation. Only much later, when this letter became known in the mathematical community, did people realize that Gauss had long before understood the principal theorem of complex function theory, which later became known through Cauchy.

Another example: when Abel, in the 1820's published articles about transcendental functions, Gauss wrote to Bessel:

> "Zur Ausarbeitung meiner seit vielen Jahren (1798) angestellten Untersuchungen über die Transcendenten Funktionen werde ich vorerst wohl noch nicht kommen können, da erst noch mit manchen anderen Dingen aufgeräumt werden muss. Herr Abel ist mir, wie ich sehe, jetzt zuvor gekommen und überhebt mich in Beziehung auf etwa ein Drittel dieser Sachen der Mühe, zumal er alle Entwickelungen mit vieler Eleganz und Consicion gemacht hat. Er hat gerade denselben Weg genommen welchen ich 1798 einschlug, daher die grosse Uebereinstimmung der Resultate nich zu Verwundern ist".

Time and time again Gauss found that other people were publishing articles on subjects which he had thought out decades ago. In mathem tics, as one of his biographers says, it was only Eisenstein who could tell Gauss something new.

But genius is not the only reason that makes Gauss unapproachable; there is a kind of reserve and reticence in his whole personality. The feeling of distance which I experienced in reading about Gauss is not caused solely by the time that separates me from Gauss, nor can I explain it entirely as my own personal reaction, for we can find this experience in the reactions of contemporaries of Gauss as well. This experience was so essential for me that I shall not try to present a direct picture of Gauss. I prefer to talk about five persons who, at different periods of his life, were very close to Gauss, and in that way I shall try to see Gauss at a distance through the eyes and reports of others.

These five persons are: Gauss' mother, a fellow student, a colleague astronomer, his daughter and finally a student.

2. Dorothea Gauss, née Benze (1743-1839) married Gebhard Dietrich Gauss in Brunswick in 1776. One year later she gave birth to a son, Carl Friedrich Gauss. Her husband had several jobs during his life, ultimately he worked as an administrator. In the early days the family was poor, later Gebhard Gauss was able to guarantee his family a moderate but secure income.

Dorothea no doubt noticed the first signs of her son's genius in the events that are recalled as anecdotes from Gauss' early years: how, for instance, when he was three, he discovered a mistake in his father's calculation of the wages for one of his servants. Another event occurred when her son was seven and attended the Katharina-school. The teacher had set the class the task of calculating the sum 1+2+3+....+100 - probably to get a bit of peace for himself. Carl saw the trick, wrote the answer on his slate and handed it in. When at last all pupils had finished the work, Carl having waited all the time with his arms crossed, his answer proved to be correct, much to the teacher's astonishment.

Often Dorothea and her husband were told that their son should study and that he should have time to read books in the evening, in stead of helping to spin flax, which was a means of supplementing the family income. Gauss' mother was more easily convinced than his father, who was a stern man, righteous but often imperious in the family. But finally he too was convinced, and Carl went to the Gymnasium in 1790. He jumped several classes. His teachers helped him to obtain financial support for his study. He took books home which he borrowed from a professor of the Collegium Carolineum in Brunswick (a kind of polytechnic,predecessor of the present technical university there). This professor furthered his study in other ways, and thus it happened that in his fourteenth year Dorothea Gauss' son was presented at the court of the duke of Brunswick. The duke, Karl Wilhelm Ferdinand, was impressed and promised to pay for the boy's further studies, first at the Collegium Carolineum (1792-1795), then at the university. And so, in 1795 Dorothea saw her son leave the house to go to Göttingen to study mathematics.

He came home at intervals, sometimes walking the whole way between Göttingen and Brunswick. Once, in 1797, he brought a friend home with him, a Hungarian fellow student. Understandably, his mother tried to find out through this friend how her son was getting on in that strange world of mathematics, books and scholars. So she took him aside and asked him how things were going and if he thought that her son would make a success of his life. He answered something like this: Madam, your son will be the greatest mathematician in Europe - and she burst out crying.

The friend was Farkas Bolyai and I shall describe the next period in Gauss' life throught the eyes and reports of this friend. That does not mean that Gauss' mother no longer played a part in his life. She lived to a very old age and she saw for herself how Bolyai's prediction came true. Gauss' father died in 1808 and after 1817 she lived with her son in Göttingen, were she died in 1839, at the age of 97.

3. Farkas (in German: Wolfgang) Bolyai (1775-1856) was two years older than Gauss. He was born in Transylvania and from 1796 he studied in Göttingen, where he became a close friend of Gauss. He was able to see sides of Gauss which his mother could not see. First of all he shared Gauss' love for mathematics. Often they talked about it, for instance about the foundations of geometry, about the parallel-postulate, which Bolyai thought he could prove, whereas Gauss by then was already almost convinced that in addition to Euclidean geometry there is a non-Euclidean geometry which has an equal right to exist. They probably also discussed the discovery which Gauss used to present himself as a mathematician to a wider audience: the proof that a regular 17-gon can be constructed with ruler and compass. Gauss here had a typical flash of inspiration as we know from a later letter in which he described the discovery:

> "Durch angestrengtes Nachdenken über den Zusammenhang der Wurzeln (namely of the cyclotomic equation) untereinander nach arithmetischen Gründen glückte es mir bei einem Ferienaufenthalt in Braunschweig am Morgen (-) diesen

> Zusammenhang auf das klarste anzuschauen, so dass ich die spezielle Anwendung auf das 17-Eck und die numerische Bestätigung auf der Stelle machen konnte".

Gauss published the proof in the **Intelligenzblatt der allgemeinen Literaturzeitung** (1796) - this was his first publication. The discovery was a turning point: it induced Gauss to opt for the study of mathematics; before that he still considered philology as a possible alternative. It was the first discovery which he noted in his **Notizenjournal** in which he subsequently (till 1814) recorded, with dates, his main discoveries.

This is not to say that he had not discovered anything before 1796; on the contrary. His studies on prime numbers date from 1792, he found the prime number theorem empirically. In 1794 he studied the arithmetic-geometric mean and its relation to elliptic integrals. In 1795 he found the method of least squares and during the same period he proved the quadratic reciprocity theorem. Later, in Göttingen, his main interest was transcendental functions and number theory.

It is not known how much Gauss told Bolyai about all this; nor is it clear how much Bolyai would have understood. But Bolyai also saw other aspects of his friend, his love of reading, his enjoyment of music and his romantic "Schwärmerei" which was common in that time in Germany. Bolyai had these feelings himself even more strongly. They experienced their friendship very strongly. They became friends on an evening of the last day of a month and they promised each other henceforth to sit down in the evening on the last day of every month and to smoke a pipe, if possible together, if not then they should think about each other. Bolyai also tells about walks he took with his friend during which they would occasionally both become so absorbed in their thoughts that they walked on for hours without speaking to each other.

In 1799 the friends parted. Bolyai went to Hungary where he became a professor at a technical college. They exchanged letters, but the intervals between the letters became longer and longer. The letters make clear what a high opinion Bolyai had of Gauss and how much their friendship had meant and still meant to him.

Bolyai was confronted with difficult family problems and he felt very isolated in Hungary. Göttingen was now far away: from the letters we learn that for years Bolyai had tried to send Gauss some bottles of wine from his own vineyards. Repeatedly the consignment had failed to arrive but finally in 1836 a crate of bottles reached Göttingen after a long detour. One third of the bottles was broken or contained bad wine. Gauss wrote back that the rest of the wine was quite good, although his mother could only drink it with sugar, but she often put sugar in French red wines anyway.

The year when Gauss and Bolyai saw each other for the last time, 1799, also marked the official end of Gauss' student period: his promotion. His thesis was about the principal theorem of algebra - that every n^{th} degree equation with real coefficients can be decomposed into linear and quadratic factors. Later he produced three other proofs of that theorem, the last one on the occasion of the fiftieth anniversary of his doctorate. The proof of the theorem which Gauss gave in his thesis is based on the use of complex numbers, in particular their geometrical representation as the complex plane, but Gauss avoids the terminology of complex numbers in the formulation of the proof. Gauss' promotion was not at Göttingen but at Helmstedt; his promotor was J.F. Pfaff (1765-1825).

In fact in Göttingen Gauss had to a great extend taught himself. He had not learned much from the professors there; his promotion at Helmstedt was largely a formality, Gauss himself was not present. At that time he was again living in Brunswick and was able to devote himself completely to scientific studies, for which the duke gave him a stipend. These studies primarily concerned number theory. He wrote the **Disquisitiones Arithmeticae** which, after many difficulties with the printer, was finally published in 1801. During these years Gauss also developed an interest in astronomy. The year 1801 in fact marks the end of Gauss' first working period, in which mathematics was his chief interest. During that first period he had his greatest inspirations and his best results, but he kept them almost completely to himself.

After that the astronomical period in Gauss' scientific work begins. It was also the period when his fame began. In 1802 he could write to Bolyai - not without some pride we can assume - that now letters addressed simply "an den Doctor Gauss, Braunschweig", reached him. 1801 also marked the beginning of a new friendship, namely with Olbers, and I shall record the next period of Gauss' life throught the eyes and reports of Olbers.

4. Heinrich Wilhelm Olbers (1758-1840), almost twenty years older than Gauss, was a physician in Bremen. He was also an amateur atronomer. In his attic he had set up a well equipped private observatory. The observatory, together with the fact that he needed very little sleep, helped him to become one of the best astronomers in Germany.

In the first weeks of the year 1801 the Italian astronomer Piazzi had observed a small celestial object which he had recognized as a hitherto unknown planet; he called it Ceres. After six weeks the planet disappeared in the twilight. This created a problem: where would the planet be when it became visible again? This required the calculation of the planet's path, but the calculation was of a new type: the data were known only over a very short period, reliable estimates of the parameters of the path (eccentricity, position of the perihelium) could not be made, hence the usual methods for calculating planetary paths were inadequate here. Gauss heard about the problem in the summer of 1801. It fascinated him and he developed a new method for calculating positions of planets, especially for this case. He published the method, together with his prediction of the position where Ceres would become visible again. This turned out to be the very position where Olbers and Von Zach found the planet. They themselves had also made estimates of the position, but these were so far wrong that they would never have led to the rediscovery of Ceres. Through this episode Gauss won a reputation as a theoretical astronomer and Gauss and Olbers became friends.

For Olbers Gauss was a young friend and colleague. Olbers greatly admired Gauss' genius; this admiration was mingled with a kind of fatherly care. In letters (the two had an intensive correspondence) Olbers expressed his admiration for Gauss' new theories on the movement of planets and for Gauss' technique of calculating the perturbances of the paths. Gauss presented his innovations in his **Theoria motuum corporum coelestium (1809)**, which is a classic of celestial mechanics; in this book he also explained the method of least squares.

Olbers also gave his friend advice about the various astronomical posts that were offered to him, for instance a post at the St. Petersburg observatory. Gauss refused the offers, because the duke was willing to augment his stipend. Meanwhile Olbers tried to get a post for Gauss at Göttingen. He was successful and Gauss was offered the post of professor of astronomy and director of the observatory. He accepted and from 1807 till his death he held these positions in Göttingen.

Olbers watched closely Gauss' rise to fame and was pleased to witness his personal happiness. From letters he learned how happy Gauss felt in his love for Johanna Osthoff, whom he married in 1805; how a son was born, and later, in Göttingen, a daughter; how Gauss enjoyed family life and how, after very few years, this happiness was suddenly shattered. After the birth of her third child, Johanna died in 1809 - and some months later the child died too. It was with Olbers that Gauss found consolation, not now through letters but personally; he went to stay with Olbers in Bremen for some weeks.

Fairly soon afterwards, in 1810, Gauss remarried; his second wife was Minna Waldeck, a friend of his first wife. The correspondence with Olbers continued, they discussed mainly astronomical topics, they exchanged observations and calculations of the paths of planets and perturbations. But now Olbers heard less about the pleasures of family life; Gauss' second marriage gave him little happiness. His enormous energy and productivity continued, but Gauss' work seemed to become more of a duty during this period. Sometimes work served as an escape from family troubles and occasionally there were signs of depression.

Olbers, and other correspondents as well, were now confronted with less easy traits of Gauss' character, his perfectionism for instance. He lived up to his motto **Pauca sed Matura** (few, but mature) in that he did not publish many of his findings, although many people, Schumacher for instance, urged him to publish his work. However, Gauss did value priority in invention over priority in publications which led to quarrels like the one with Legendre about the method of least squares. Also Gauss' repeated assertions that results published by others had already been known to him long before, created resentment. Later it became clear that these assertions were always justified; but it is understandable that mathematicians,especially the younger ones, sometimes doubted this at the time.

Gauss appeared sensitive and touchy - a friendship with Bessel came to an end for that reason. Other people, particularly Bolyai's son Janós were grieved by Gauss' incomprehensible attitude towards his own genius. Janós Bolyai had developed a theory of non-Euclidean geometry in the 1820's, which, through his father, had reached Gauss. Gauss later told others that he considered Janós Bolyai to be a mathematical genius of the first rank. But in his reply to Bolyai we read:

> "Jetzt einiges über die Arbeit Deines Sohnes. Wenn ich damit anfange dass ich solche nicht loben darf : so wirst Du wohl einen Augenblick stutzen: aber ich kann nicht anders; sie loben hiesse mich selbst loben: denn der ganze Inhalt der Schrift, der Weg den Dein Sohn eingeschlagen hat, und die Resultate zu denen er geführt ist,kommen fast durchgehends mit meinen eigenen, zum Theile schon seit 30 - 35 Jahren angestellten Meditationen überein".

Janós strongly resented this refusal of recognition and the claim of priority, and that is quite understandable. Another example: Jacobi, who was always a great admirer of Gauss, sometimes complained that Gauss had never cited himself or Dirichlet in his publications for many years.

The contact with Olbers was centred on Gauss' astronomical work. There were two other scientific friends who had a similar relation to Gauss: Schumacher and Weber.

Heinrich Christian Schumacher (1780-1850), the astronomer, prepared and executed together with Gauss a geodetic measure and research programme, namely the triangulation of Hannover, connected with the triangulation of Schleswig-Holstein, then ruled by the King of Denmark. In connection with this programme Gauss did a lot of field work in the years 1821-1825; he travelled much, often in difficult circumstances, supervising or performing measurements himself; it was quite an extensive project. It also inspired theoretical work on geodesics and differential geometry, in particular Gauss' **Disquisitiones generales circa superficies curvas** which was published in 1827.

After the astronomical period, involving the contact with Olbers, and the geodetic period with Schumacher, came a physical period, also involving a colleague and friend: Wilhelm Weber (1804-1891). In this case there was no correspondence but direct collaboration at Göttingen, where Weber, through Gauss' mediation, had become professor of physics in 1831. They worked together on magnetism. Again this inspired Gauss to produce theoretical studies on the subject, notably **Die allgemeine Theorie des Erdmagnetismus** (1838) and the **Allgemeine Lehrsätze in Beziehung auf die in verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungskräfte** (1839).

5. Olbers died in 1840. By then Gauss had been a widower for nine years; his mother had also died. The children from his first and second marriage had left home, except for his daughter Therese. Therese lived with her father till his death; it was she who set the atmosphere in the house in the last 15 to 20 years of his life. She was very dear to him.

Therese Gauss was born in 1816, the third child of the second marriage. She must have had a rather unhappy childhood. In 1820 it became clear that her mother had tuberculosis - it was this that caused her death in 1831 and in the last years the mother suffered much. She also was slightly neurotic. And Therese's father, Gauss, was not an easy man. Towards his sons especially he was hard and he showed little understanding. In the case of

Eugen, the first son of the second marriage, this resulted in a break. In 1829 Eugen left home and went to America, after a big row about his misbehaviour as a student. Gauss was hurt; to Olbers he wrote about his son calling him the "Taugenichts", who caused himself and his ill wife so many worries.

Also Gauss' attitude to his own genius was involved in his relation with his sons. He did not want them to study mathematics - for which both sons of the second marriage had some talent - because they would then be in the shadow of their father's genius, or mediocre work would be published under the name of Gauss.

After these episodes the family was confronted with problems, arising from politics. Already earlier Gauss' life had been directly influenced by political events; in 1806 his benefactor, the duke, died from wounds received in battle as commander in chief of the army that tried to stop Napoleon's invasion. Germany was defeated, the Germans had to pay reparations, Gauss himself was required to pay 2000 francs. Finally an anonymous admirer paid the sum for him, but after that period Gauss' feelings for France and the French were decidedly cool.

In Göttingen, life was not always quiet either. The town was regarded as a centre of revolutionary activities. In 1830/31 there had been demonstrations against the privileged position of the nobility. Partly through these protests, reforms were instituted in 1833. But in 1837 the new and very autocratic king of Hannover, Ernst August, annulled these reforms at one stroke. This led to protests, the best known of which was the declaration of the "Göttingen Sieben"; seven Göttingen professors openly protested against the revocation of the reforms. Among the seven was Weber. Another of them was Von Ewald, professor of Eastern languages and Gauss' son in law, he was married to Gauss' daughter Wilhelmine. The seven were promptly sacked; Von Ewald and two others had to leave the country at once, Weber and the three others were allowed to stay but were forbidden to express their views in public. Protests had no effect, the king declared that he had enough money to buy as many ballet girls, whores and professors as he liked.

The affair affected the Gauss family personally. Should Gauss join the seven? Many expected that he would, or that at least he would protest against the sacking of his son in law, his friend and others. He did neither. It is difficult to find explanations for his silence. Gauss himself gave as a reason the fact that he had to look after his aged mother. But he was also unsympathetic to revolutionary movements and politically he was conservative. Gauss' political ideal was formed through his relation to the duke: a state under an enlightened ruler. In his youth it had been precisely this system that had given him the opportunity to develop his gifts.

Gauss certainly suffered as a result of the affair of the Göttingen seven; his collaboration with Weber ceased, as did his contact with his daughter Wilhelmine, who followed her husband into exile. Gauss became very lonely during this period - Olbers died some years after the affair. In this time of loneliness Therese was a great comfort to him, especially when, after 1850, he also experienced physical infirmities. Gauss suffered from insomnia. And he developed a heart condition, which finally caused his death in 1855. Shortly after his death Therese described, in a very touching letter to her brother, how close she and her father had been in these last years.

6. Through Gauss' mother and daughter we have glimpsed something of his personal life, through Bolyai we have seen his studies and through Olbers and others his scientific work. We have not heard much about his activities in teaching and administration. He performed these tasks dutifully but did not enjoy them. He taught astronomy and many of his students became great astronomers; there has been a Gauss-school of astronomy. As far as mathematics was concerned things were different. Only in the 1850's did Gauss come to enjoy teaching mathematics to some students. One of them was Riemann, another Dedekind. Dedekind later described Gauss' lectures very vividly; he tells us:

> "Gauss trug ein leichtes, schwarzes Käppschen, einen ziemlich langen braunen Gehrock, graue Beinkleider. Er sass meist in bequemer Haltung, etwas gebeugt vor sich

> niedersehend, mit über den Leib gefalteten Händen. Er sprach ganz frei, sehr deutlich, einfach und schlicht. Wenn er aber einen neuen Gesichtspunkt hervorheben wollte, wobei er ein besonderes charakterisch-tisches Wort gebrauchte,so erhob er wohl plötzlich den Kopf, wandte sich zu seinem Nachbarn und blickte ihn während der nachdrücklichen Rede ernst mit seinen schönen durchdringlichen blauen Augen an. Das war unvergesslich....
> Ging er von einer prinzipiellen Erörterung zur Entwicklung mathematischer Formeln über, so erhob er sich, und in stattlicher, ganz aufrechter Haltung schrieb er an einer neben ihm stehenden Tafel mit der ihm eigenen Handschrift, wobei es ihm immer durch Sparsamkeit und zweckmässige Anordnung gelang mit dem ziemlich kleinen Raume auszukommen. Für die Zahlen-beispiele, auf deren sorgfältige Durchführung er besonderen Wert legte, brachte er die erforderlichen Daten auf kleinen Zetteln mit".

7. And so, although I have not been able to sit at Gauss' table, I have been able through Dedekind, to show you his lecture room. I have tried, respecting the distance which I feel so strongly, to look at Gauss and to tell you something about his life and work. The emphasis has been on his life, his relations with others, their reactions and Gauss' attitude towards his own genius. I hope that, despite the distance, Gauss has become a closer and more familiar figure to you all.