Assignment shortconceptual due 02/01/2018 at 11:59pm EST

---

1. (1 point) Thinking of determinants as areas of parallelograms, in what terms are these rules best interpreted?

[?]1. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & b-a \\ c & d-c \end{vmatrix}$

[?]2. $\begin{vmatrix} k\cdot a & b \\ k\cdot c & d \end{vmatrix} = k\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

A. reshaping a parallelogram like a stack of books
B. a similar but scaled parallelogram
C. the perimeter of the parallelogram
D. reshaping a parallelogram like four sticks
E. introducing a third dimension/thickness
F. base times (perpendicular) height formula
G. stacking parallelograms side to side

---

2. (1 point) A pizza is to be shared among x friends. They cut it into so many equal pieces. Then one more friend shows up. The pizza now has to be divided into $x+1$ pieces, but the cutting had already taken place. Therefore each person cuts off an xth piece of their slice and give it to the newcomer. How much smaller did each piece of pizza become? This illustrates the fact that the derivative of $f(x) =$ _____ is $f'(x) =$ _____ Does everyone have the same amount of pizza in the end? [/yes/no]

---

3. (1 point) Logarithms have two key properties. Originally they were important in a computational context because they "turn multiplication into addition": $\ln(ab) = \ln(a) + \ln(b)$. In the context of the calculus they are central because of the differentiation rule $\ln'(x) = 1/x$. Let us see how these two are related. To do this, we shall start with the second property and see if we can connect it to the first.
Using

-
- FTC1
- FTC2
- product rule
- chain rule
- quotient rule

, we see that the function $f(x) = \int_c^x \frac{1}{t}dt$ has the derivative $f'(x) =$ __________
For the formula $f(ab) = f(a) + f(b)$ to hold for all positive numbers a and b, it is necessary that $f(p) = 0$ for a certain number p. Explain why. (Hint: Consider simple values for a and b.) $p =$ _____. Therefore, $c =$ _____.
How is the derivative of $f(ax)$ related to the derivative of $f(x)$?

- $f'(ax) = f'(x)$
- $f'(ax) = -f'(x)$
- $f'(ax) = af'(x)$
- $f'(ax) = f'(x)/a$
- None of the above

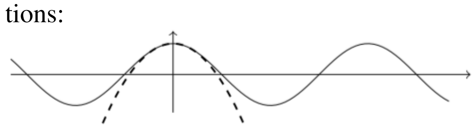
What does this imply about how $f(ax)$ is related to $f(x)$? Use what your found regarding p to further specify this relation. $f(ax) =$ __________

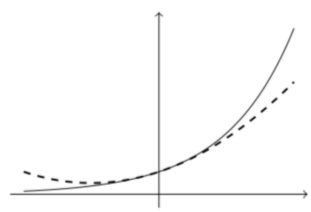
---

4. (1 point) By picturing the graph of $\ln(x)$, I feel that the power series for $\ln(1+x)$ starts with a [/zero/positive/negative] constant term, a [/zero/positive/negative] linear term, and a [/zero/positive/negative] quadratic term.

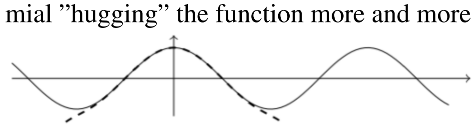
---

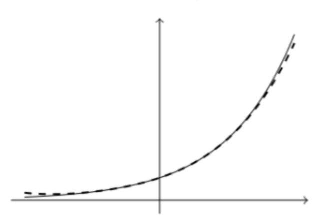
5. (1 point) If we cut a power series off after the second-degree term we get the best possible parabolic approximation. Here I have illustrated this for the cosine and exponential functions:





If we include more terms of the series we will see the polynomial "hugging" the function more and more closely, like this:





Suppose I use the first five terms of the power series for e^x to approximate $e^{0.1}$, then use this result to find an approximation for e by

-
- raising the result to the power 10
- taking 1 divided by the result
- multiplying the result by 10
- taking the ln of the result and multiplying by 10

. Alternatively, I could find an approximation for e directly from the series by

-
- plugging in x=0
- plugging in x=1
- using a geometric series
- using a binomial series

. Which of the two methods will be more accurate?

-
- the first
- the second
- both equal

---

6. (1 point) If $z = f(x,y)$ is the roof of a building, in what direction will rain water flow?

-
- grad f
- - grad f
- perpendicular to grad f
- along contour curve of f

---

7. (1 point) Consider two lines, $y = ax + b$ and $y = cx + d$, that intersect in some point. The vectors (___ $,a)$ and (___ $,c)$ point in the direction of these lines respectively. If the lines are perpendicular, the product of their slopes is ___.

---

8. (1 point) $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$ if and only if **a** and **b** are perpendicular. One can see this geometrically by means of

-
- the Pythagorean Theorem
- Theorem of Thales
- diagonals of parallelograms
- trigonometry

.
To prove it algebraically, it is useful to consider:

- A. $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})$
- B. $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$
- C. $(\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$

---

9. (1 point) In a constrained optimisation context, if $f(x,y)$ is profit as a function of money spent on television and print advertisements, what is the meaning of $g(x,y) = c$? Knowing this and solving for λ in the Lagrange equations suggests that λ has the real-world interpretation:

- profit as fraction of spending
- spending as fraction of profit
- extra dollars earned per extra dollar spent
- extra dollars spent per extra dollar earned
- payoff of TV ads relative to print ads
- payoff of print ads relative to TV ads
- increase in cost of ads with increasing spending

---

10. (1 point) The standard form for a second-order differential equation is $\ddot{x} + b\dot{x} + cx = f(t)$. In terms of a pendulum, b is air resistance, c is gravity, and $f(t)$ is forcing (i.e., an external force pushing the pendulum). By picturing this prototype example we can get a good feeling for the behaviour of such a differential equation. Use this way of thinking to associate the following differential equations with their corresponding scenario and type of solution. Enter two letters for each answer, such as Aa.

A = pendulum with no air resistance. B = child on swing pushed well by parent. C = child on swing given out-of-synch pushes. D = pendulum in syrup. E = air "encouragement" instead of air resistance. F = "negative gravity." G = pendulum with slight air resistance. H = pendulum pulled in one direction.

a = perpetual oscillations. b = dying oscillations. c = growing oscillations. d = slow approach to equilibrium. e = running off to infinity. f = jerky motion.

_____ $\ddot{x} + x = 0$
_____ $\ddot{x} + x = t$
_____ $\ddot{x} + 2x = \sin(t)$
_____ $\ddot{x} + x = \sin(t)$
_____ $\ddot{x} + \dot{x} - x = 0$
_____ $\ddot{x} + 0.1\dot{x} + 2x = 0$
_____ $\ddot{x} - 0.1\dot{x} + 2x = 0$

---

11. (1 point) I have solved a second-order differential equation and found the homogenous solution x_h and the particular solution x_p. Which of the following must be true?

- A. $x_h + x_p$ is a solution to the differential equation.
- B. $x_p + 5$ is another particular solution.
- C. The choice of particular solution is not unique.

- D. Once I have an initial condition such as $x(0) = 2$ I can give one concrete answer for the general solution.

- E. $5x_p$ is another particular solution.

- F. x_h contains two undetermined constants.

- G. x_h is a solution to the differential equation.

- H. x_p is a solution to the differential equation.

- I. None of the above

---

12. (1 point) Is $a = 0.99999\ldots$ the greatest number smaller than 1? [/yes/no] Hint: consider $10a - a$.
Generalise your argument to find a closed formula for $1 + x + x^2 + x^3 + \cdots =$ __________

---

13. (1 point) Aristotle discusses the following the paradox of motion (Physics, 239b11): "[Zeno] asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal," and then the half-way stage of what is left, etc., ad infinitum.

Or as the Aristotle-commentator Simplicius (1013.4) put it: "An object in motion must move through a certain distance; but since every distance is infinitely divisible the moving object must first traverse half the distance through which it is moving, and then the whole distance; but before it traverses the whole of the half distance, it must traverse half of the half, and again the half of this half. If then these halves are infinite in number, because it is always possible to halve any given length, and if it is impossible to traverse an infinite number of positions in a finite time ... [then] therefore it is impossible to traverse any magnitude in a finite time."

This paradox is related to:

-
- series for exponential function
- series for logarithm
- geometric series
- binomial series

---

14. (1 point)
Which of the following have a common root meaning with the mathematical term *integral* ?

- A. We need to integrate immigrants into society.

- B. Common name for 1, 2, 3, ...

- C. Spaghetti integrale.

- D. None of the above