Mathematics for Poets

Content. In this course we take a big-picture look at the nature of mathematics and its role in human thought, emphasising its interactions with society, history, philosophy, science, and culture. The course involves studying key mathematical arguments in some detail, but our goal is not to develop a repertoire of technical skills. Instead, we study accessible mathematical topics specifically selected for their rich interconnections with cultural context, and integrate our mathematical work with reflections on its broader meaning and implications. Instead of the drill and practice problems of a traditional mathematics class, we approach mathematics through seminar discussions, hands-on activities, and readings connecting it to broader issues. We thus analyse a selection of emblematic and important mathematical proofs and use them as a platform for reflecting on the nature of mathematics. In parallel, we read excerpts from seminal historical texts across the ages as well as modern scholarship from a wide range of academic disciplines that shed light on the interplay between mathematics and its societal and intellectual context. We focus especially on geometry, from the origins of mathematical reasoning in early civilizations, to Euclid's Elements that was the gold standard of exact reasoning for millennia and the model for countless philosophical systems, to the projective geometry of Renaissance art, to the more modern non-Euclidean geometry that overturned conventional wisdom about the nature of human spatial perception and the shape of space.

Aims. After completing the course students are able to:

- Discuss critically, and situate in scholarly and historical context, reflections on the nature of mathematics.
- Draw on a range of examples to discuss the role of mathematics in society, culture, and human thought.
- Read simple mathematical proofs and explain the ideas and methodologies involved.
- Reason rigorously and conceptually about fundamental notions in a simple geometric setting.
- Relate fundamental aspects of mathematical reasoning to its broader meaning and purpose.
- Discuss the nature of mathematical knowledge and axiomatic-deductive systems, especially on the basis of key foundational concepts of Euclidean and non-Euclidean geometry.

Format. We interleave mathematical topics with seminar discussions of a rich array of short readings that connect the material to a broader cultural, philosophical, and historical context. We study a selection of mathematical proofs and topics drawn from geometry with an emphasis on conceptual understanding. To this end we supplement textual mathematical sources with physical models and hands-on activities that allow us to experience and explore geometry in a concrete way.

Assessment.

25% Midterm exam

25% Final exam

Midterm and Final exams will consist of 7 questions, of which you answer 4 of your choosing. The questions are selected from the seminar discussion questions printed before each section of the reader, and the general Euclid discussion questions.

40% Assignments

Assignments are the questions recognisable by the □ symbol in the course notes.

10% Participation

Participation means active engagement in discussions. Criteria for an A grade:

- For each Seminar class, you come prepared with a question or comment that you think could make for interesting discussion, in addition to at least one good bullet point in reply to each of the assigned discussion questions.
- For each Euclid class, you have studied the assigned proofs attentively so that you know all the key steps without having to consult the text, and you have looked at the assigned discussion questions and have at least an idea or question to get you started toward collectively resolving the questions in class.
- In discussions, you make substantive connections across different readings and materials in the course, or with other relevant background knowledge you may have.
- In discussions, you make an effort to understand the points of view of others and engage with them thoughtfully.

Course calendar. For Seminar and Euclid classes, it is essential that you come to class prepared, having carefully studied and thought about the assigned readings and looked at the associated questions. For Activity classes, look at the corresponding section of the notes in advance and read the "story" part of it; the problems and activities we will then tackle together in class.

Day		Topic	Reading
1	Activity	Geometry on surfaces	§1
2	Activity	450 degree cone	§2
3	Euclid	foundations; ruler and compass	*Elements* –3; §E1
4	Seminar	Greek geometry and philosophy: beginnings	§R1
5	Euclid	triangle congruence	*Elements* 4–8; §E2
6	Activity	Geometry on a sphere	§3
7	Seminar	Textual aspects of Greek mathematics	§R2
8	Euclid	further basic constructions	*Elements* 9–11; §E3
9	Activity	Ruler and compass, Angle trisection	§4, §5
10	Seminar	Greek geometry and philosophy: classical age	§R4
11	Euclid	angles	*Elements* 13–16; §E4
12	Seminar	Descartes, Hobbes	§R8, §R9
13	Seminar	Do mathematical proofs explain?, Leibniz	§R6, §R10
14	Euclid	more triangle congruence	*Elements* 22–26; §E5
15	Activity	Descartes, Catenary	§6, §7
16		Midterm Exam	
17	Seminar	Perspective art, Math and society in early modern Europe	§R4, §R5
18	Activity	Perspective art	§8
19	Euclid	parallels	*Elements* 27–32; §E6
20	Seminar	Rationalism versus empiricism, Absolute versus relative space	§R11, §R12
21	Activity	Projective geometry, Erlanger Programm	§9, §10
22	Euclid	parallelograms, area	*Elements* 34–41; §E7
23	Activity	Abstract axiomatics	§11
24	Seminar	Kant, Kepler	§R13, §R7
25	Euclid	Pythagorean Theorem	*Elements* 46–48; §E8
26	Seminar	Space and perception	§R14
27	Activity	Hyperbolic geometry	§12
28	Seminar	Non-Euclidean geometry	§R15
29	Activity	Cultural history of trigonometry	§13
30		Final Exam	